System Dynamics: Building Models

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Logistic Growth

- A special case of S-shaped growth is known as logistic growth or Verhulst growth (first developed in 1938)

- In this model, the net fractional growth rate, *g*, is a linear function of the population.

$$g = g^*\left(1 - \frac{P}{C}\right)$$

- The net growth rate is then:

  g*: maximum fractional growth

$$\frac{dP}{dt} = gP = g^*\left(1 - \frac{P}{C}\right)P$$

$$= g^*P - g^*\frac{P^2}{C}$$

- The logistic model can be represented in a non-linear analytic expression:

$$P(t) = \frac{C}{1 + \left[\frac{C}{P_0} - 1\right]e^{-g^* t}}$$

Spread of Infectious Diseases

- The cumulative number of cases follows an S-shaped curve
- The rate of new cases rises exponentially, reaches a peak and then falls
- Epidemics can begin with a single patient – patient zero
- The disease spreads through contact with infected person
- As more and more people get infected, more people catch the disease
- The rate of new infection falls when the pool of uninfected people decreases

SI Model: Spread of Infectious Disease

- Total population is divided into two categories: susceptible to disease (S) and infected with disease (I)
- As people are infected, they are removed from the stock 'S' and are included in the stock 'I'
- Assume that people stay infected indefinitely
- Infectious disease spreads when a susceptible person comes in contact with an infected person. The rate of infection is I_R
- People in the community interact at a certain rate, the contact rate 'c'
- The total number of people in the community is fixed, and denoted as 'N'.
- If infected people are not quarantined, and interact at the same rate as others, the probability that any random encounter will be with an infected individual will be I/N
- The infectivity, 'i', of the disease is the probability (chance) that a person after coming in contact with an infected individual will contract the disease.

Mathematical Model

Susceptible Population: S [$people$]

Infected Population: I [$people$]

Total Population: $N = S + I \quad [people]$

contact frequency: $c \quad [persons/person/day] = [1/day]$

Susceptible contacts $= Sc \quad [people/day]$

Probability of contact between infected and uninfected person $= \dfrac{I}{N}$

Contacts between infected and uninfected people $= Sc \cdot \dfrac{I}{N}$

infectivity: i

Infection rate: $I_R = \dfrac{dI}{dt} = Sc \cdot \dfrac{I}{N} \cdot i \quad [people/day]$

Ref: Sterman, 2000

Mathematical Model

$$\text{Total Population}: N = S + I \quad [people]$$

$$\text{Infection rate}: I_R = \frac{dI}{dt} = Sc \cdot \frac{I}{N} \cdot i \quad [people/day]$$

$$\frac{dI}{dt} = (N - I)c \cdot \frac{I}{N} \cdot i$$

$$\frac{dI}{dt} = ciI\left(1 - \frac{I}{N}\right)$$

$$\frac{dI}{dt} = \boxed{ciI} \boxed{- ci\frac{I^2}{N}}$$

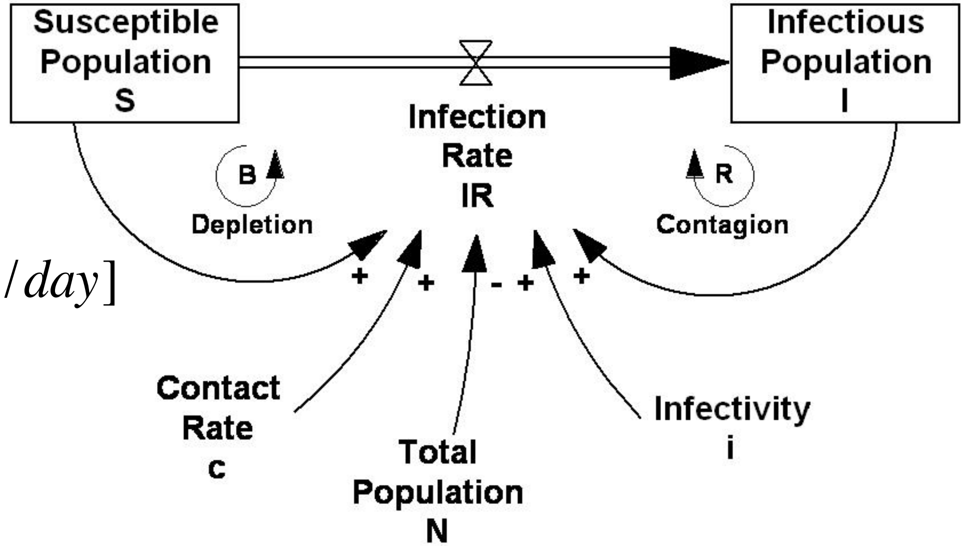
reinforcing loop
(exponential behavior)

balancing loop
(goal-seeking behavior)

Ref: Sterman, 2000

Stocks and Flow Modeling : Spread of Infections – SI Model

$$\text{Total Population}: N = S + I \quad [people]$$



$$\text{Infection rate}: I_R = \frac{dI}{dt} = Sc \cdot \frac{I}{N} \cdot i \quad [people/day]$$

$$\frac{dI}{dt} = (N - I)c \cdot \frac{I}{N} \cdot i$$

$$\frac{dI}{dt} = ciI\left(1 - \frac{I}{N}\right)$$

$$\frac{dI}{dt} = ciI - ci\frac{I^2}{N}$$

Key assumptions:

Total Population is constant (no migration, births, deaths etc)
No recovery – patients infected indefinitely
Constant contact rate

Key implication:
A single infected individual causes everyone in the community to contract the disease

Figure Source: Sterman, 2000

Technology Adoption “Epidemic”

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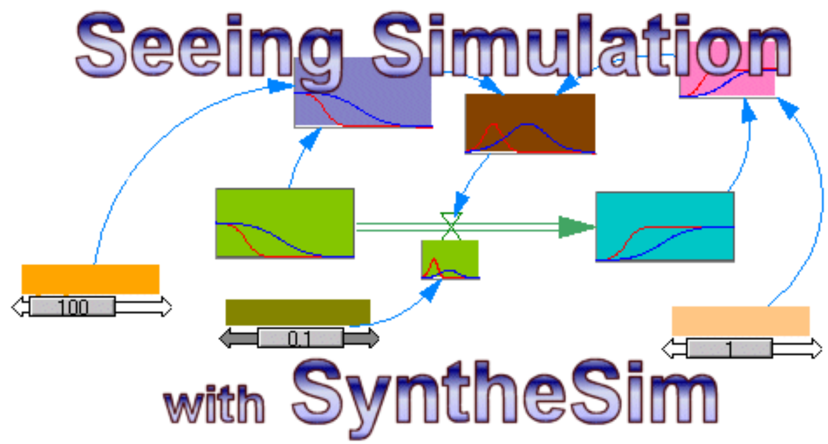

New: Vensim 5.11a, Vensim-Forio video on web deployment of models, native Macintosh support, Courses, Polish Documentation, Spanish Documentation, Vensim distributor in China, Molecules Version 2.02, Portuguese Documentation, Distance Program, Applications

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Tools for creating CLDs

Use to set polarity

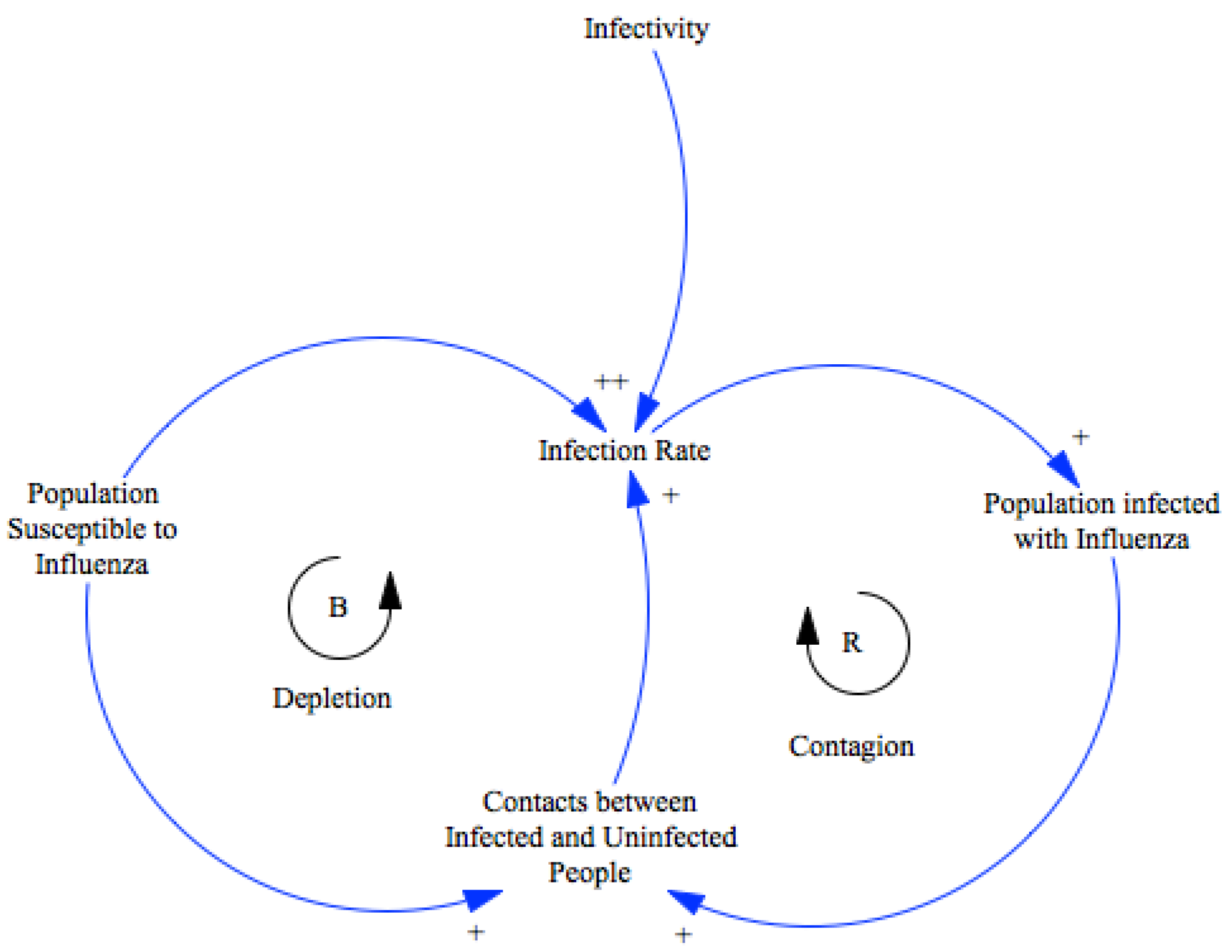
delete

Create a variable

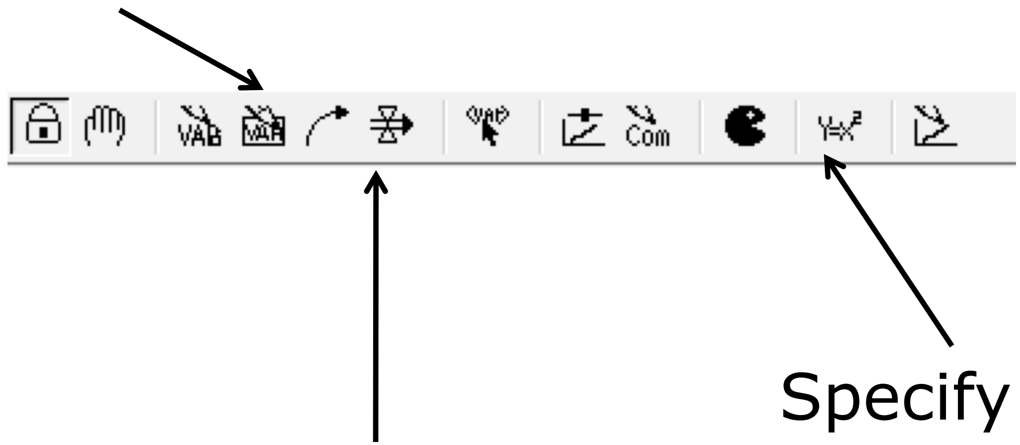
Create a causal link

Add comment
(loop identifier)

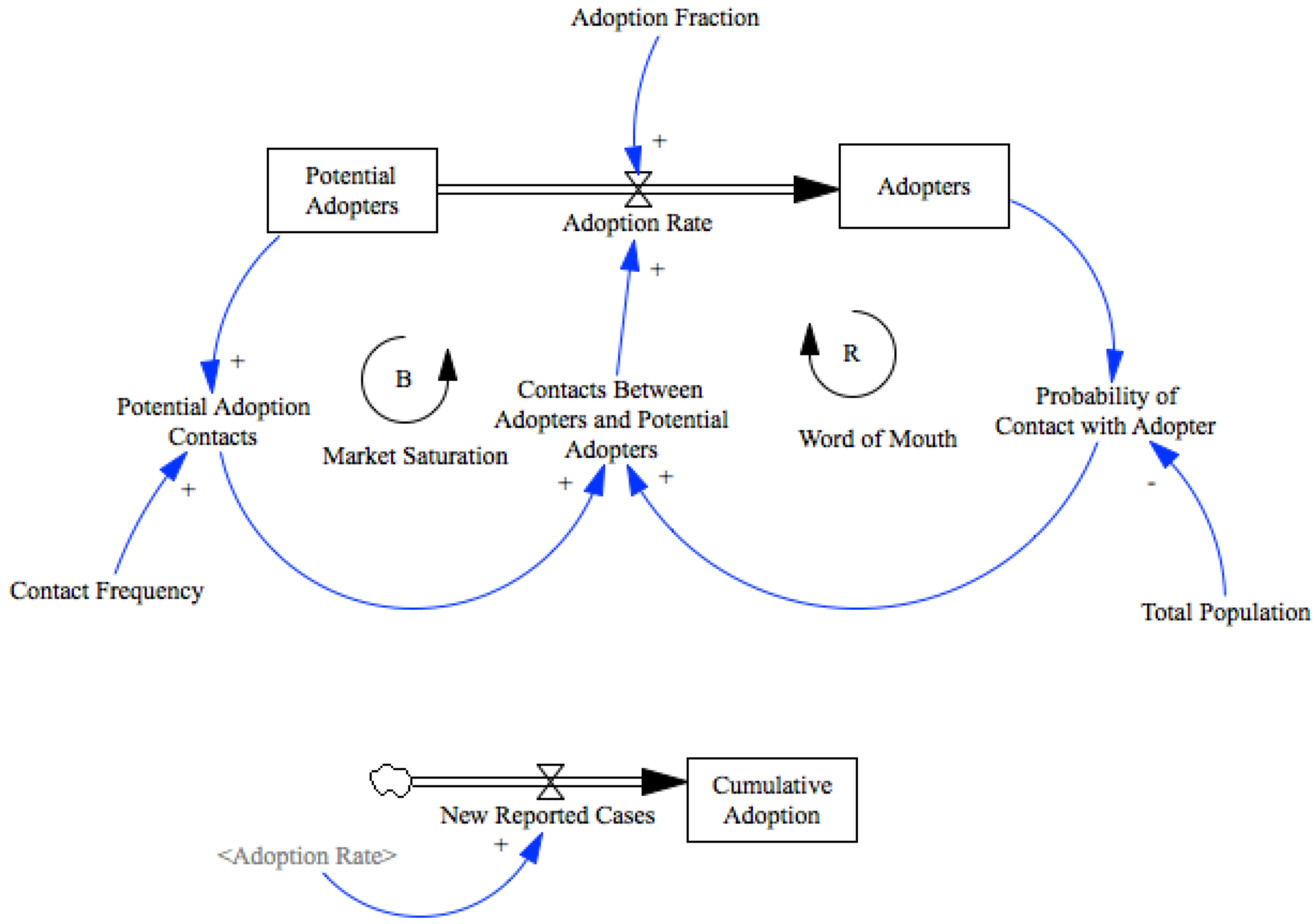
Causal Loop Diagram



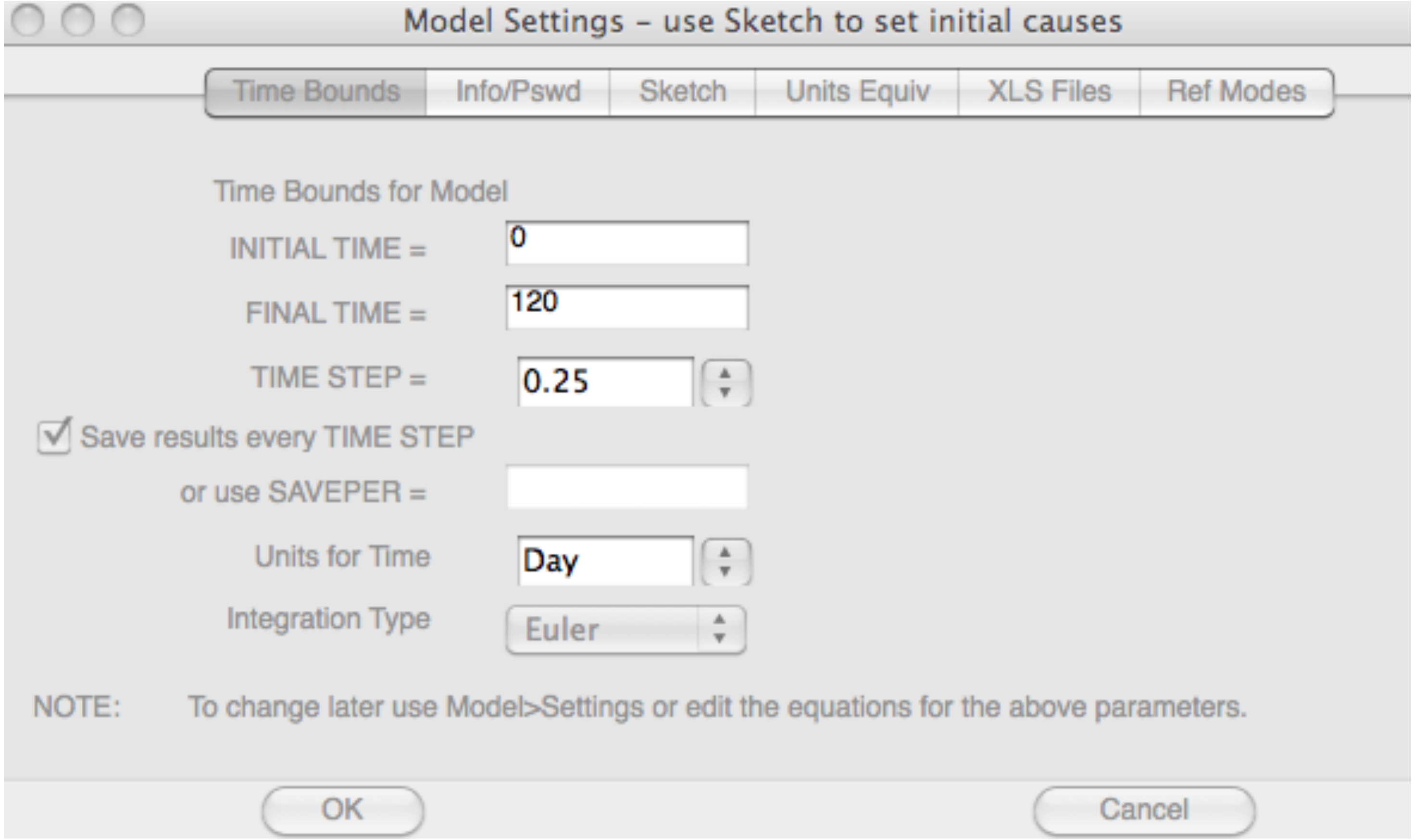
Tools for Creating Stocks and Flows



Stocks and Flow Diagram



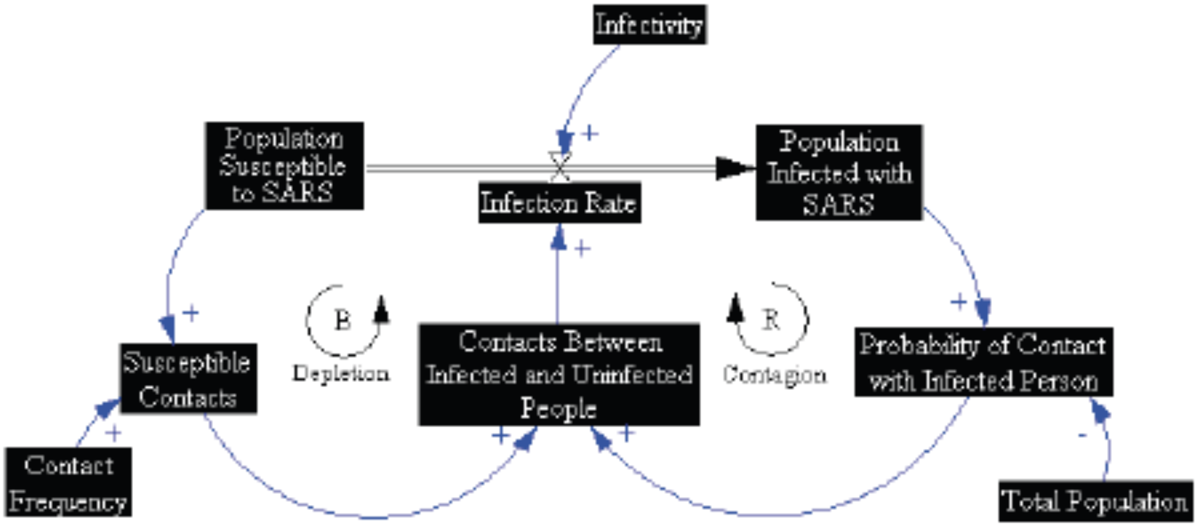
Model Settings



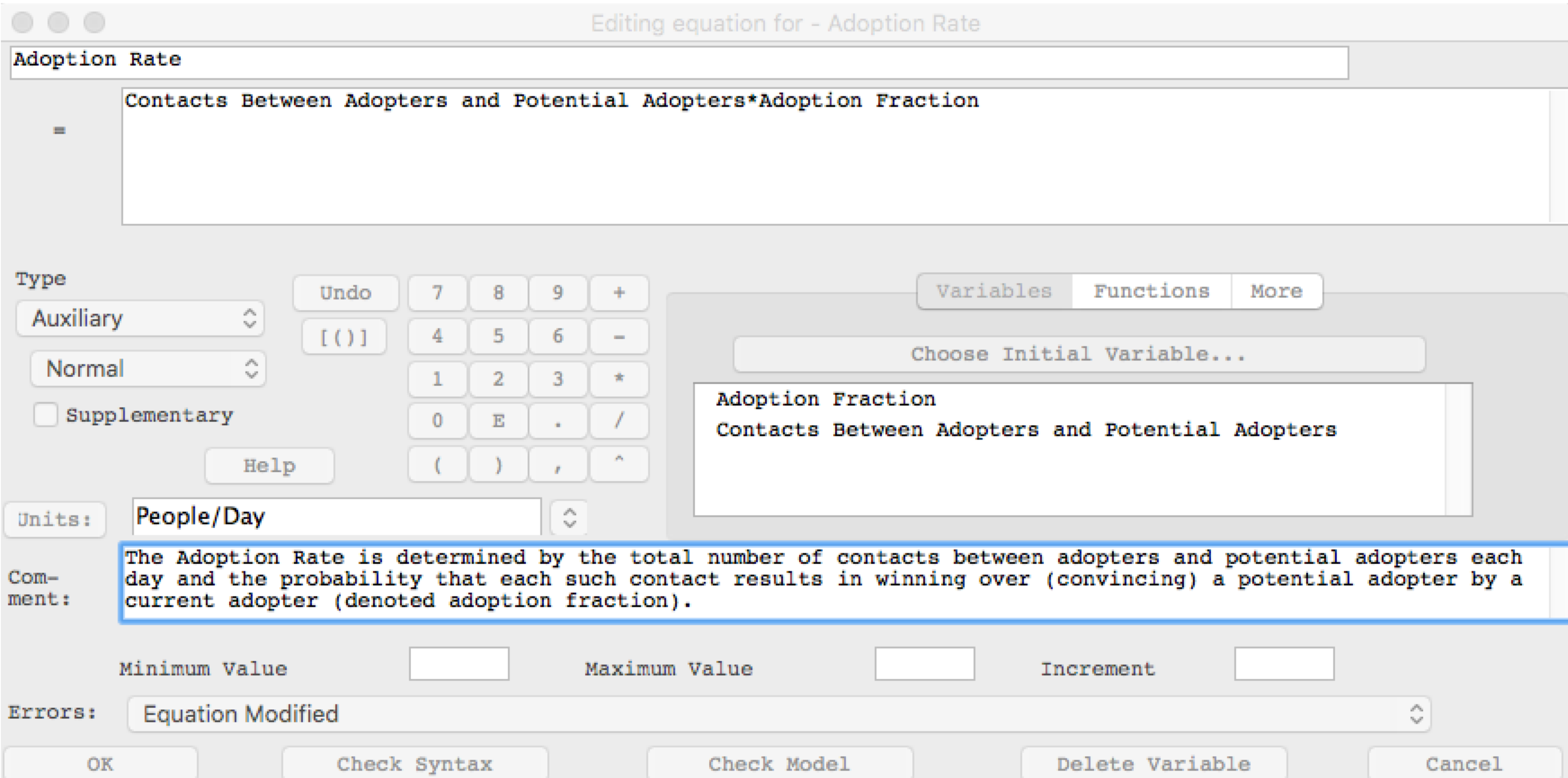
Add Equations

Specify equation

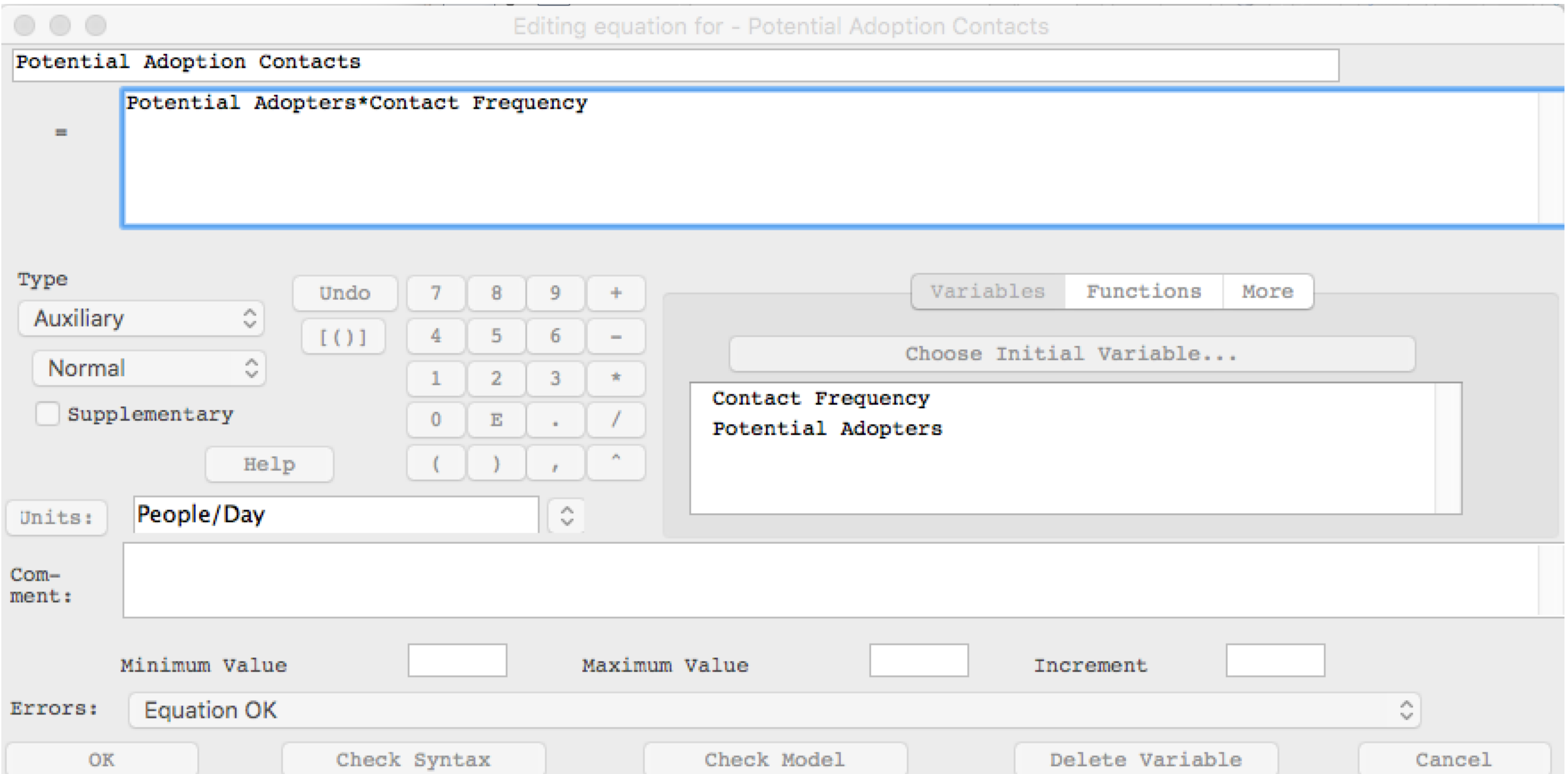
Highlighted variable means equation is incomplete



Specify Equations



Equations



Editing equation for - Potential Adoption Contacts
Potential Adoption Contacts
=
Potential Adopters*Contact Frequency
Type
Auxiliary
Normal
Supplementary
Undo
[( )]
Help
7 8 9 +
4 5 6 -
1 2 3 *
0 E . /
( ) , ^
Variables
Functions
More
Choose Initial Variable...
Contact Frequency
Potential Adopters
Units:
People/Day
Com-
ment:
Minimum Value
Maximum Value
Increment
Errors:
Equation OK
OK
Check Syntax
Check Model
Delete Variable
Cancel

Equations

- Contacts Between Adopters and Potential Adopters =

Potential Adoption Contacts*Probability of Contact with Adopter

Units: [People/Day]

- Probability of Contact with Adopter = Adopters/Total Population

Units: [Dimensionless]

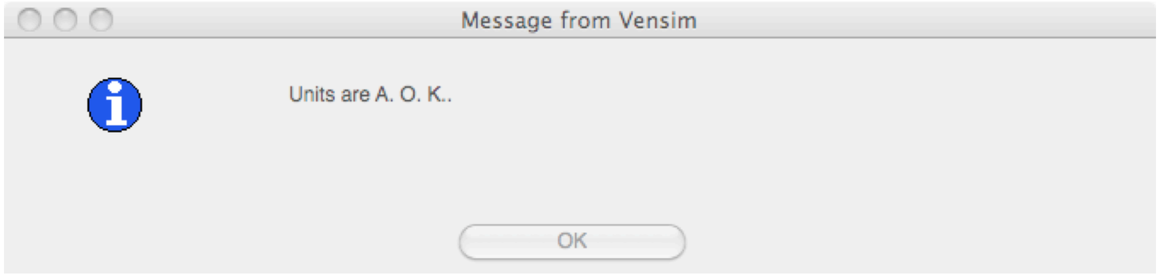
- Total Population = 350 [People]
- Contact Frequency = 1 [People/day]
- Adoption Fraction = 0.15 [Dimensionless]
- Adopters : Initial Condition? [units?]
- Potential Adopters = Total Population [units?]

Check Model

- Model>> Units Check

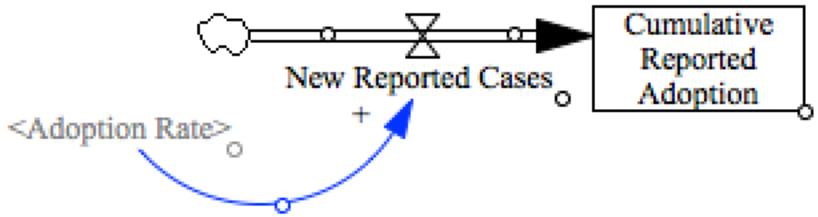
- If an error appears, check all equations to make sure they are dimensionally consistent.
- Contact Frequency is # of people/person/day: [1/day]

- Correct units or equations until on more errors remain.

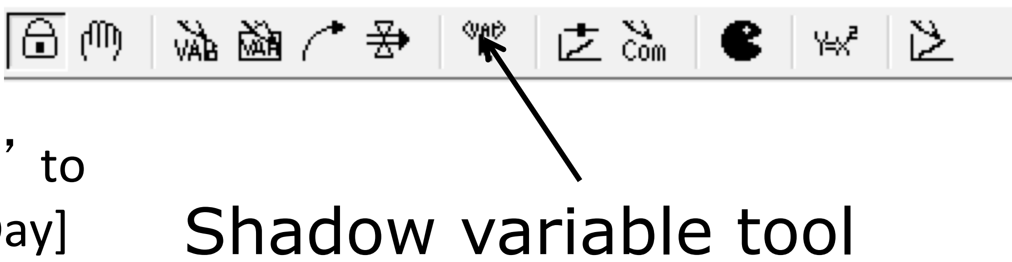


Create a Shadow Variable

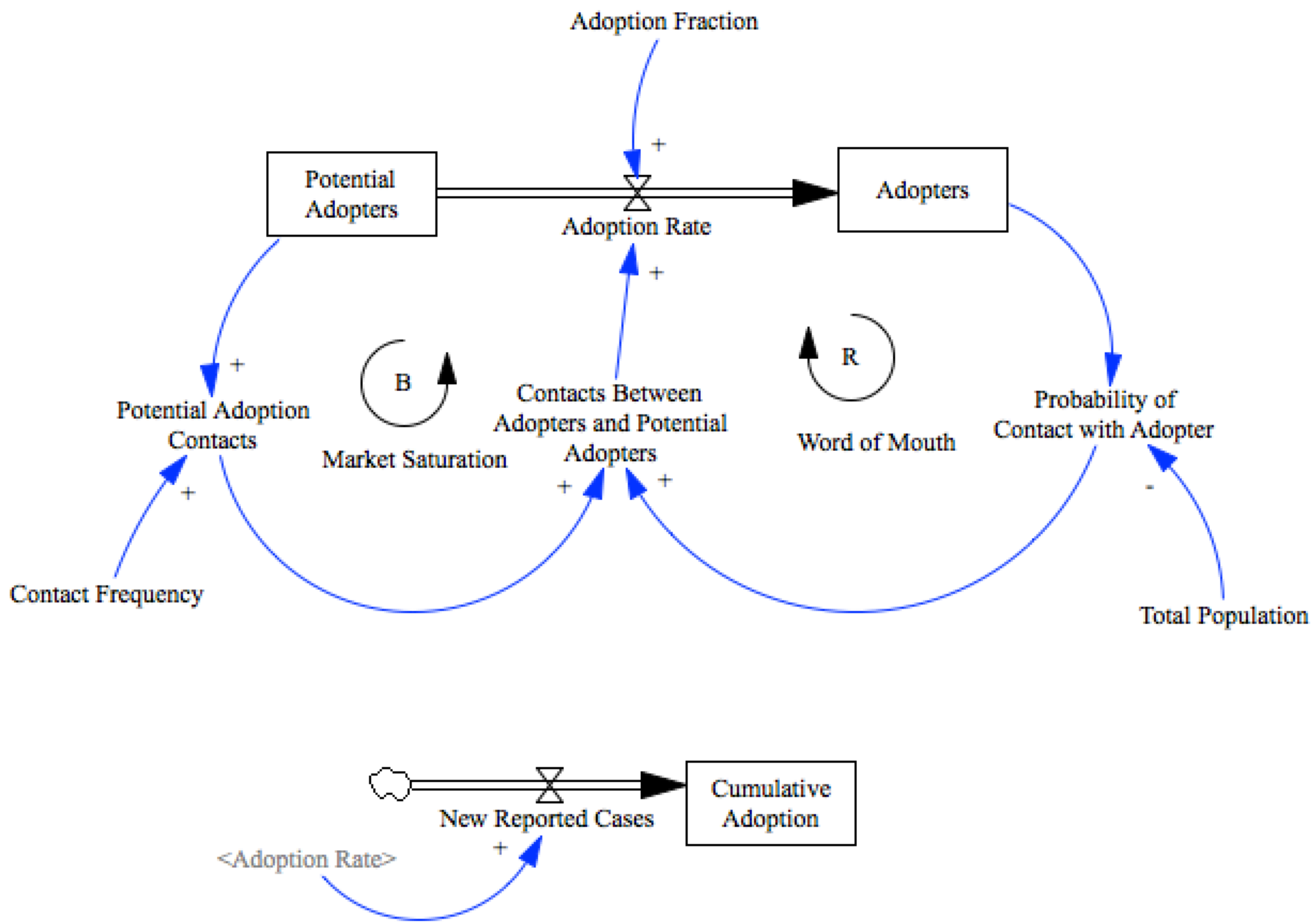
- Create the stock and flow structure shown:



- Make 'New Reported Cases' equal to 'Adoption Rate' (in reality they may not be equal due to reporting delays, errors etc.) by creating a Shadow variable of Adoption Rate



- Set equation of 'New Reported Cases' to Adoption Rate. Set Units to [People/Day]


Adoption Fraction
+
Potential
Adopters
Adoption Rate
Adopters
+
+
B
R
Potential Adoption
Contacts
Market Saturation
Contacts Between
Adopters and Potential
Adopters
Word of Mouth
Probability of
Contact with Adopter
+
+
+
-
Contact Frequency
Total Population
New Reported Cases
Cumulative
Adoption
<Adoption Rate>
+

Simulate Model

- Type in a name for the simulation run. Call it 'Base' (instead of 'current').

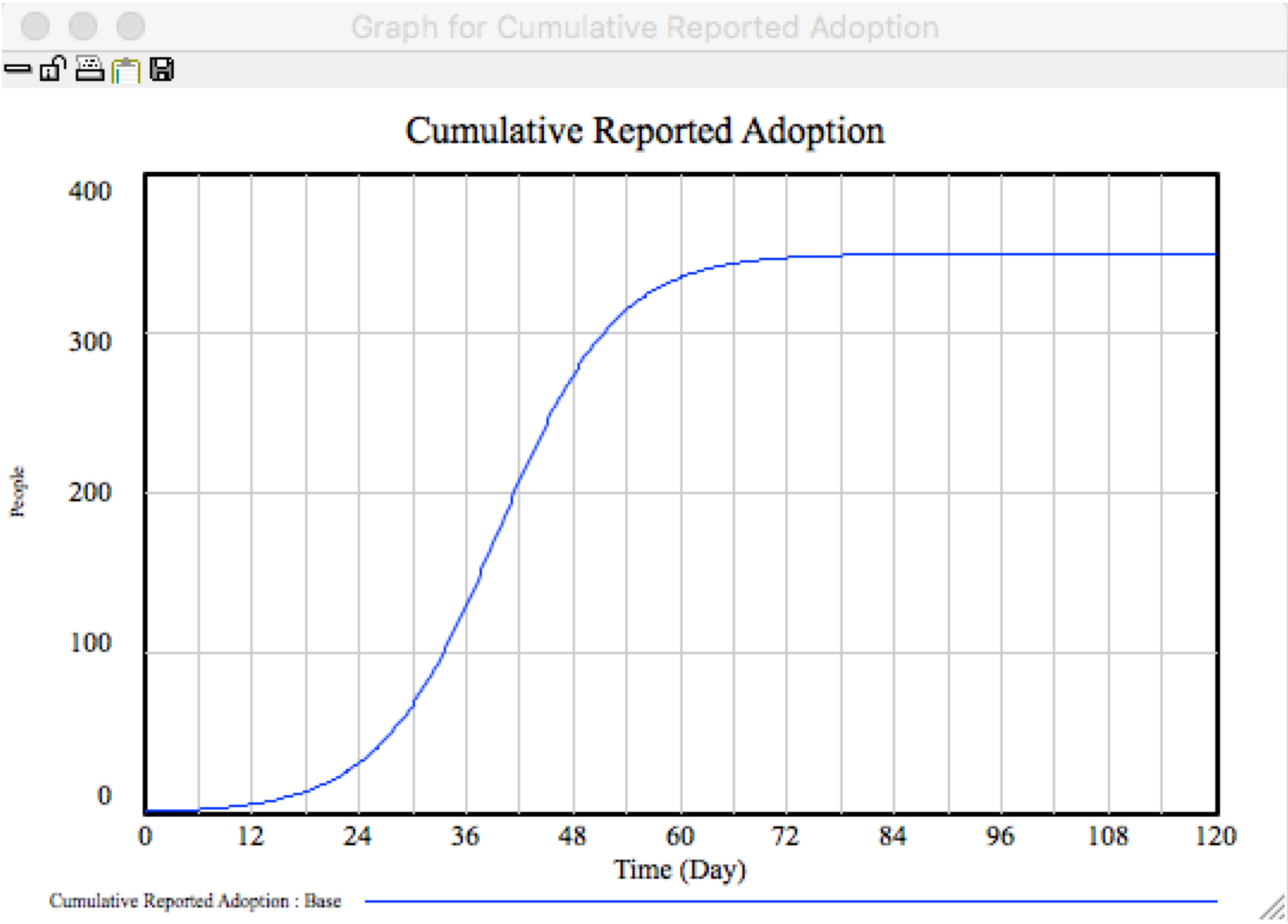
Run Simulation

- To see a graph, click on 'hand tool', click on variable to see, then click on graph button.

Hand tool

Graph tool

Graph Results

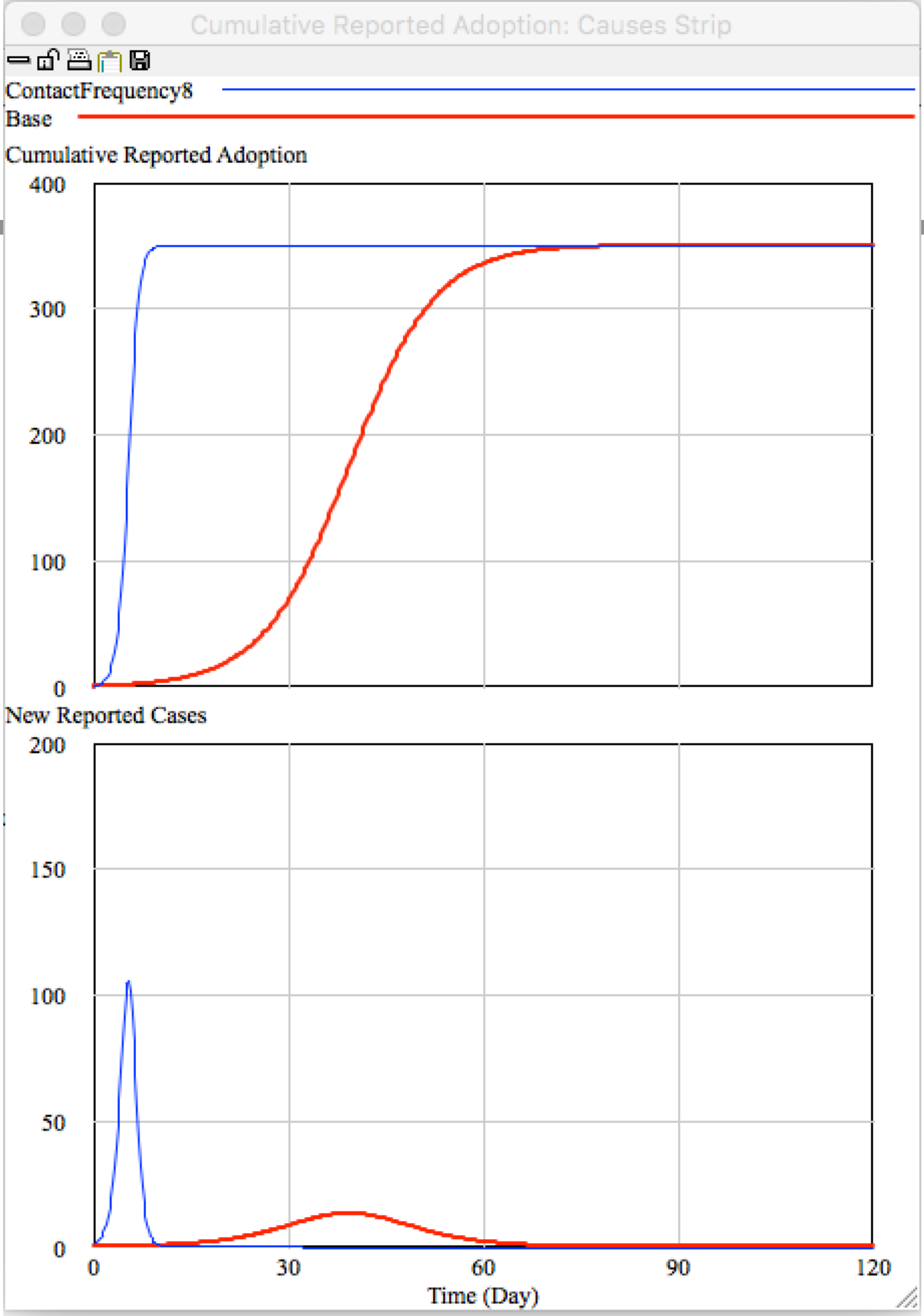


Other Results Analysis Tools

- To see a graph, click on ‘hand tool’, click on variable to see, then click on graph button.

Graph tool for all variables
Determining selected variable

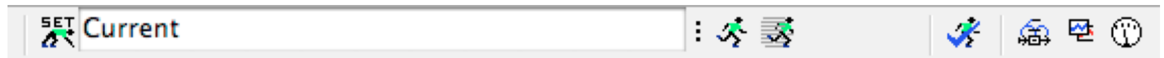
Tabular Data



Cumulative Reported Adoption: Causes Strip
ContactFrequency8
Base
Cumulative Reported Adoption
400
300
200
100
0
New Reported Cases
200
150
100
50
0
0
30
60
90
120
Time (Day)

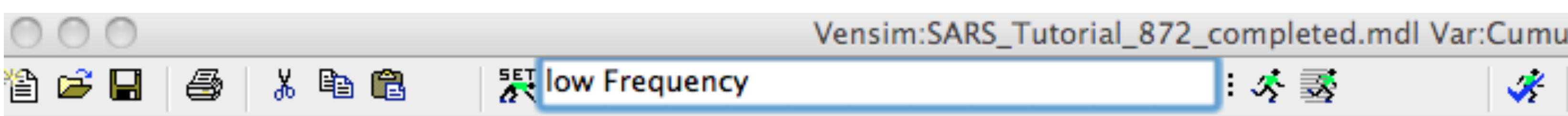
Test Effect of Parameters

- Click the ‘Set’ button



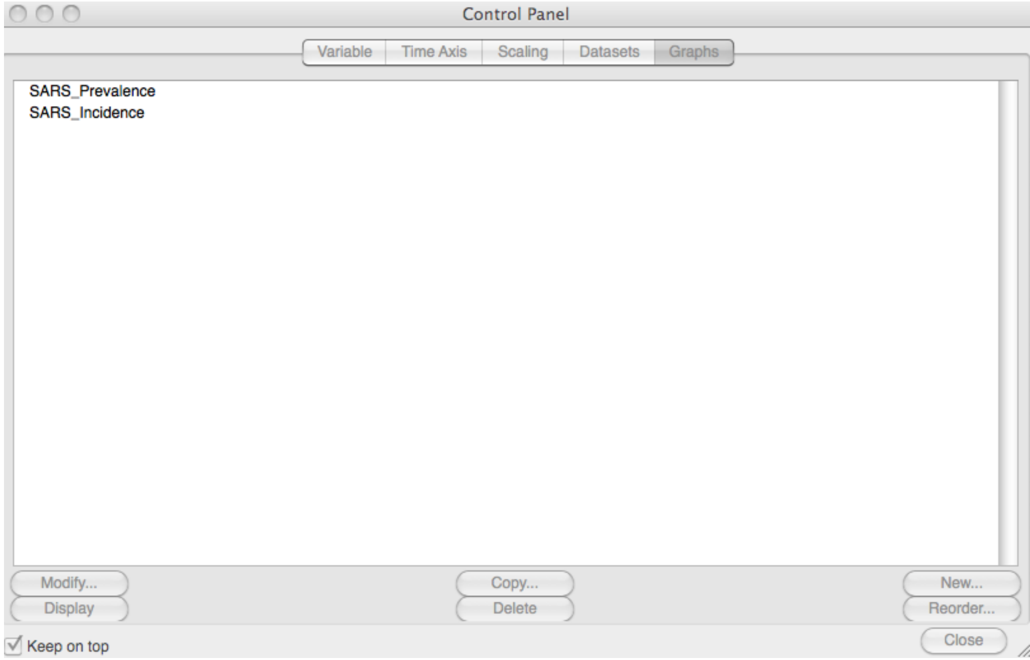
- Click on ‘Contact Frequency’ and then change value. Change name of simulation-run e.g. ‘low contact frequency’.

- Simulate and check results

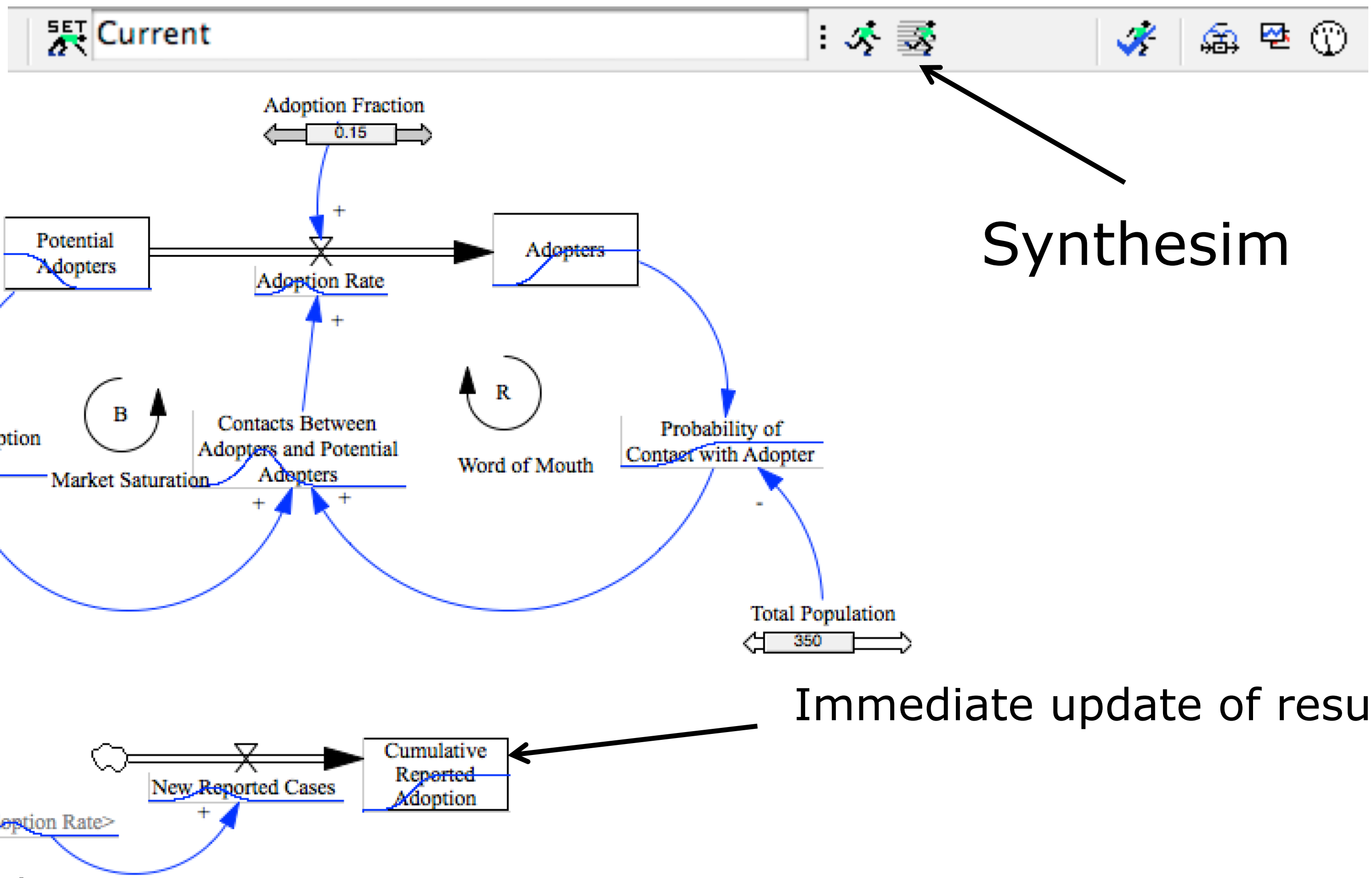


Custom Graphs

- Click 'Control Panel' button
- Select 'Graphs' tab and customize graph
- Select 'Datasets' and unload data sets that you do not want to see plotted in charts



Quick Tests: Synthesim



Synthesim

Immediate update of results

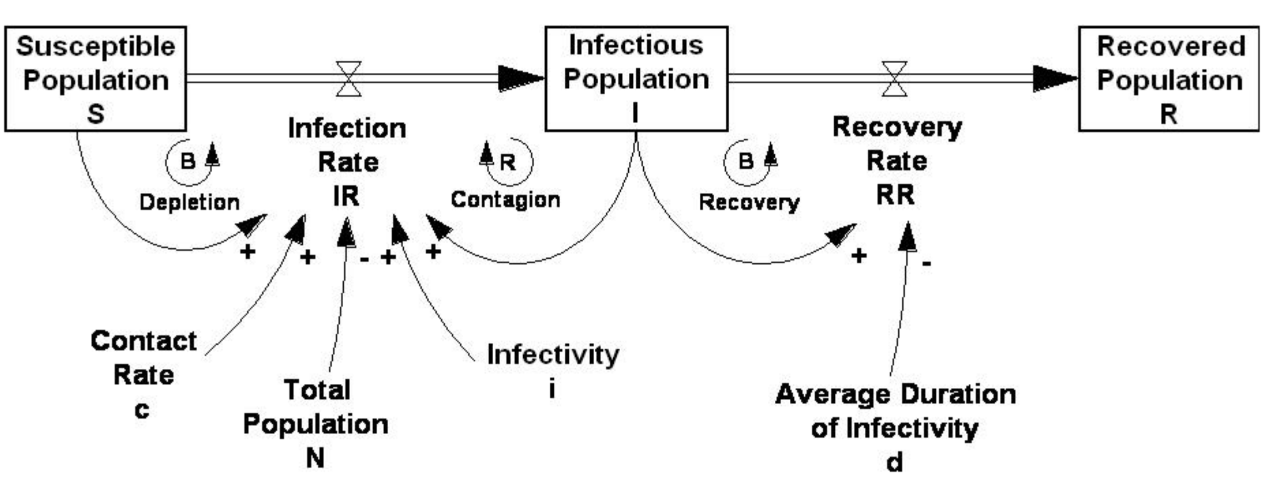
Slider bar to change values

Stocks and Flow Modeling : Spread of Infections – SIR Model

$$S = \left(N - I_o - R_0\right) + \int_{t_0}^{t_f} -I_R \, dt$$

$$I = I_o + \int_{t_0}^{t_f} \left(I_R - R_R\right) dt$$

$$R = R_o + \int_{t_0}^{t_f} R_R \, dt$$



$$\text{Recovery rate}: R_R = \frac{I}{d} \quad [people/day]$$

d: average length of time people are infectious

Greater the number of infected individuals, greater the recovery rate and then lower the total number of infected individuals – we get a balancing loop

Key assumption:
Patients remain sick for limited time then recover and develop immunity

Key implication:
Some people may not contract the disease

Figure Source: Sterman, 2000

The Tipping Point

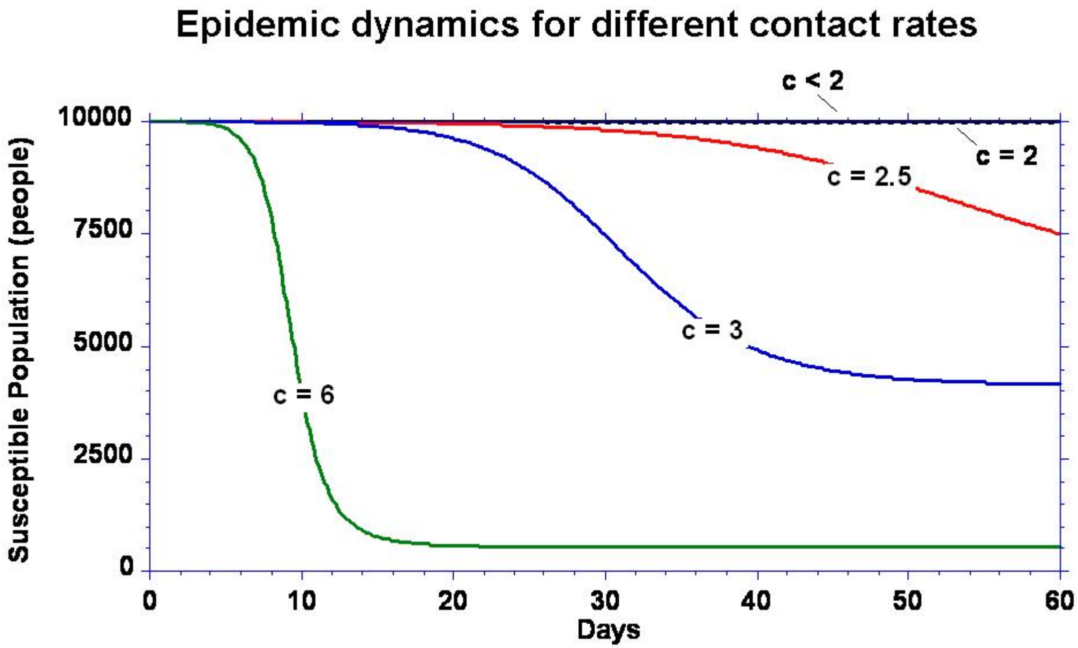
- The SIR model is second-order (two independent stocks)

- Unlike the SI model, the disease may die out without causing an epidemic – if recovery rate is faster than infection rate, infectious population will fall causing infection rate to fall. It may go to zero before everyone catches the disease.

- When does an epidemic occur?

Depends on which loop is dominant

- Below the tipping point, the system is stable.
- Above the tipping point the disease can spread very fast and is only limited by the depletion of the susceptible stock.



N=10,000 people, i =0.25, d=2 days
$I_o = 1$, $S_o = N$

Figure Source: Sterman, 2000

Model Validation

- The key factor influencing the acceptance and success of models is their practical usefulness.
- A model is useful when it serves the purpose for which it was developed: it addresses the right problem at the right scale and scope.
- Models are an abstraction of reality, and the greater the level of uncertainty and complexity of the problem, the more superficial objective comparisons between predicted results and observed data become.
- Model validation is a social process where model structure and outcome are negotiated until judged valid and useful by all involved parties
- Model usefulness requires transparency of the model development process and the model itself.

Some Rules to Model By:

- Develop a model for solving a problem
  - Model should have clear purpose, do not include extraneous factors
  - Start simple, add details as necessary over time
- Approach model with skepticism
  - Model is not reality (only a limited abstraction)
- Use other tools and data
  - Effective models use data and empirical analysis
- Model should be developed iteratively and jointly with stakeholders
  - Avoid black boxes, build understanding and trust
- Validate with continuous testing, iteration, and stakeholder input

Limitations of Systems Dynamics Modeling - 1

- Due to the uncertainties inherent in complex open systems, SDM will not provide exact solutions and answers. It is thus not suited to address well-defined operational problems

- Differences in value judgements can dramatically influence which policies are ultimately recommended

- Definition of the problem boundary, i.e. the model breadth, can be problematic. Modellers should only include variables if they contribute to generating the problem behaviour as experienced in reality

- *System dynamics modelling is more of an art than a science*

Limitations of Systems Dynamics Modeling - 2

- System dynamics primarily aids analyses of complex interdisciplinary problems that inherit uncertainty.

- Attempts to resolve operational problems are best addressed using other methods.

- Given its flexibility and transparency, and increasing recognition of the complex, multi-dimensional nature of water resource management issues, SDM provides valuable tools for analysis and foresighting, guiding decision-making.

- Key factors that have assisted these developments have been the capacity to integrate qualitative and quantitative information, the ability to integrate a wide range of input parameters in a meaningful way (reflecting their inherent interactions and feedbacks),

Summary Remarks

- Prospects for success are maximised when the group itself constrains the definition of the problems to be addressed, and participatory procedures are applied in scoping, development and testing of the model.
- Involvement underpins ownership, providing the platform for management applications that are not only responsive to group concerns, but also have greater prospects for effective implementation and uptake
- SDM provides an opportunity to meaningfully test projections of system futures and the reliability/deficiencies in our understanding. In light of these insights, we are able to progressively adapt management strategies to changing circumstances.
- Such flexibility is vital for prevailing development pressures, climate change and measures that deal with the long term consequences of our past “command-and-control” legacies.

I. Winz, G. Brierley and S. Trowsdale, Water Resour Manage (2009) 23:1301–1323

Additional Information

Vensim can be downloaded from:

http://vensim.com

Systems Dynamics Society

http://www.systemdynamics.org

Systems Dynamics Conference

https://conference.systemdynamics.org

Systems Dynamics Review (journal)

http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1099-1727

References

- Business Dynamics: Systems Thinking and Modeling for Complex World, Sterman, McGraw Hill (2000).

- I. Winz, G. Brierley and S. Trowsdale, “The Use of System Dynamics Simulation in Water Resources Management”, *Water Resources Management* (2009) 23:1301–1323

- V. Tidwell, H. Passell, S. Conrad, and R. Thomas, “System dynamics modeling for community-based water planning: Application to the Middle Rio Grande”, Aquatic Sciences, (2004), 66: 357–372